**Solution to Problem 4**) Considering that $x$ and $y$ are independent random variables, and that $z = x - y$, we may write

$$\langle z \rangle = \langle x \rangle - \langle y \rangle. \tag{1}$$

$$\langle z^2 \rangle = \langle (x-y)^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle - 2\langle xy \rangle = \langle x^2 \rangle + \langle y^2 \rangle - 2\langle x \rangle \langle y \rangle. \tag{2}$$

Consequently,

$$\sigma_z^2 = \langle z^2 \rangle - \langle z \rangle^2 = (\langle x^2 \rangle + \langle y^2 \rangle - 2\langle x \rangle\langle y \rangle) - (\langle x \rangle^2 + \langle y \rangle^2 - 2\langle x \rangle\langle y \rangle) = \sigma_x^2 + \sigma_y^2. \tag{3}$$

The same results can be obtained by starting with the characteristic function $\psi_z(s)$ of the random variable $z$. Since, for the random variable $-y$, the probability density function is $p_y(-y)$, the corresponding characteristic function is found to be

$$\psi_{-y}(s) = \int_{-\infty}^{\infty} p_y(-y) \exp(-\mathrm{i}2\pi sy)\, \mathrm{d}y = \int_{-\infty}^{\infty} p_y(y) \exp(\mathrm{i}2\pi sy)\, \mathrm{d}y = \psi_y^*(s). \tag{4}$$

Consequently, the characteristic function $\psi_z(s)$ of $z = x - y = x + (-y)$ is given by the product $\psi_x(s)\psi_y^*(s)$. As expected, $\psi_z(0) = \psi_x(0)\psi_y^*(0) = 1$. As for the derivatives of $\psi_z(s)$, we have

$$\psi_z'(s)|_{s=0} = \left[\psi_x'(s)\psi_y^*(s) + \psi_x(s){\psi_y'}^*(s)\right]_{s=0} = \psi_x'(0) + {\psi_y'}^*(0). \tag{5}$$

$$\begin{aligned}\psi_z''(s)|_{s=0} &= \left[\psi_x''(s)\psi_y^*(s) + \psi_x(s){\psi_y''}^*(s) + 2\psi_x'(s){\psi_y'}^*(s)\right]_{s=0} \\ &= \psi_x''(0) + {\psi_y''}^*(0) + 2\psi_x'(0){\psi_y'}^*(0).\end{aligned} \tag{6}$$

Comparison with Eq.(29) of Sec.7 now reveals that $\langle z \rangle = \langle x \rangle - \langle y \rangle$, and $\langle z^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle - 2\langle x \rangle\langle y \rangle$. These, indeed, are the same results that we obtained directly in Eqs.(1) and (2).